The Astrophysical Journal, 597:L169–L172, 2003 November 10

TRAPPING OF SOLAR ENERGETIC PARTICLES BY THE SMALL-SCALE TOPOLOGY OF SOLAR WIND TURBULENCE

D. Ruffolo,[1,2] W. H. Matthaeus,[3] and P. Chuychai[1]
Received 2003 July 8; accepted 2003 September 19; published 2003 October 17

ABSTRACT

The transport of energetic particles perpendicular to the mean magnetic field in space plasmas has long been viewed as a diffusive process. However, there is an apparent conflict between recent observations of solar energetic particles (SEPs): (1) Impulsive solar flares can exhibit "dropouts" in which the SEP intensity near Earth repeatedly disappears and reappears, indicating a filamentary distribution of SEPs and little diffusion across these boundaries. (2) Observations by the *IMP-8* and *Ulysses* spacecraft, while they were on opposite sides of the Sun, showed similar time-intensity profiles for many SEP events, indicating a rapid lateral diffusion of particles throughout the inner solar system within a few days. We explain these seemingly contradictory observations using a theoretical model, supported by computer simulations, in which many particles are temporarily trapped within topological structures in statistically homogeneous magnetic turbulence and ultimately escape to diffuse at a much faster rate.

Subject headings: diffusion — magnetic fields — Sun: particle emission — turbulence

1. INTRODUCTION

In general, energetic particles in space plasmas gyrate in helical orbits around magnetic field lines, and transport parallel to the mean magnetic field is more rapid than perpendicular transport (Parker 1963, p. 242). In particular, the interplanetary magnetic field is dragged outward from the Sun in a spiral pattern by the solar wind (Parker 1958), and the particles accelerated by violent events near the Sun (such as solar flares and coronal mass ejections) can rapidly travel to the observer when there is a good magnetic connection between them. While spatial inhomogeneities in solar energetic particle (SEP) distributions have been known for decades, they have generally been reported as occasional, sharp features attributed to magnetic discontinuities, including shocks, magnetic sector boundaries, tangential discontinuities, fast/slow solar wind boundaries, large-scale flux tubes, and magnetic clouds (e.g., Scholer & Morfill 1975; Evenson, Meyer, & Yanagita 1982; Dröge, Wibberenz, & Klecker 1990; Sanderson et al. 2000). However, the "dropouts" recently observed by the *Advanced Composition Explorer* (*ACE*) spacecraft for a large number of impulsive solar events occur so frequently and over such small scales (∼0.03 AU) that they cannot be attributed to large-scale features but instead must be related to the small-scale structure of the interplanetary magnetic field (Mazur et al. 2000). Indeed, we argue that dropouts are a signature of the topology of magnetic turbulence in the solar wind and therefore are relevant to understanding magnetohydrodynamic turbulence in general.

SEPs from impulsive solar events serve as a good probe of lateral transport (in solar latitude and longitude) because they arise from a localized source (Reames, Cane, & von Rosenvinge 1990) associated with a group of sunspots. Lateral transport requires transport perpendicular to the mean magnetic field as a function of time, which is in turn attributed to the random walk of turbulent field lines as a function of distance along the mean field (Jokipii 1966). That substantial lateral transport occurs over a timescale of days is dramatized by recent observations of the same set of solar events by two spacecraft: the *Interplanetary Monitoring Platform 8* (*IMP-8*) near Earth and *Ulysses* at 2–2.8 AU (McKibben, Lopate, & Zhang 2001). The inferred magnetic footpoints of the two spacecraft were nearly opposite in solar longitude and also very different in solar latitude (equatorial for *IMP-8* and near the South Pole for *Ulysses*). Those authors presented time-dependent SEP fluxes that were very similar (in absolute terms) during the decay phases of the majority of the observed events. While in at least one of their events there was an abrupt change near the time of a shock passage, heralding the onset of temporally and spatially invariant spectra in the region downstream of the shock (e.g., McKibben 1972), some profiles were diffusive with no apparent shock passage and simply converged at late times. This indicates that SEPs can undergo rapid lateral diffusion, spreading throughout the inner solar system within a few days.

However, perpendicular transport of a diffusive nature cannot explain both the dropouts and the *IMP-8*/*Ulysses* observations; the latter, as well as previous multispacecraft observations (e.g., Palmer 1982 and references therein), imply such rapid diffusion that the small-scale dropouts would be washed out. It has been proposed that fluid motions at the solar surface lead to a field line random walk that is consistent with the dropouts (Giacalone, Jokipii, & Mazur 2000). As will be shown below, that type of random walk is too slow to explain the *IMP-8*/*Ulysses* observations. Here we propose to reconcile these observations in terms of a two-component model of solar wind turbulence that has provided a useful explanation of both its magnetic statistics and the parallel transport of SEPs. We show that in such a model, a certain fraction of low-energy SEPs is temporarily trapped within small-scale topological structures in statistically homogeneous turbulence, ultimately escaping to diffuse at a much faster rate. This view of perpendicular transport can explain both the *ACE* observations of dropouts, over short timescales, and the *IMP-8*/*Ulysses* observations of rapid dispersion after a few days.

[1] Department of Physics, Chulalongkorn University, Bangkok 10330, Thailand; david@astro.phys.sc.chula.ac.th, piyanate@corona.phys.sc.chula.ac.th.
[2] Current address: Department of Physics, Faculty of Science, Mahidol University, Rama VI Road, Bangkok 10400, Thailand.
[3] Bartol Research Institute, University of Delaware, 217 Sharp Laboratory, Newark, DE 19716; yswhm@bartol.udel.edu.

2. ENSEMBLE AVERAGE STATISTICS

We consider a two-component model of solar wind turbulence as follows:

$$\boldsymbol{B} = B_0\hat{\boldsymbol{z}} + \boldsymbol{b}^{\rm slab}(z) + \boldsymbol{b}^{\rm 2D}(x, y), \quad (1)$$

$$\boldsymbol{b}^{\rm slab} \perp \hat{\boldsymbol{z}}, \quad \boldsymbol{b}^{\rm 2D} \perp \hat{\boldsymbol{z}}. \quad (2)$$

This assumes a constant (or slowly varying) mean magnetic field plus two components of transverse fluctuations. The "slab" component of turbulence $\boldsymbol{b}^{\rm slab}$ depends only on z, the coordinate along the mean field, while the "two-dimensional" component $\boldsymbol{b}^{\rm 2D}$ depends only on the perpendicular coordinates, x and y. The two-component model was motivated by the observation that solar wind fluctuations are concentrated at nearly parallel and nearly perpendicular wavenumbers (Matthaeus, Goldstein, & Roberts 1990). Furthermore, this model provides a good explanation of the parallel transport of SEPs (Bieber et al. 1994; Bieber, Wanner, & Matthaeus 1996; Dröge 2000), providing a solution to the long-standing discrepancy between theoretical and observed scattering mean free paths.

For the two-dimensional component, we can write

$$\boldsymbol{b}^{\rm 2D}(x, y) = \boldsymbol{\nabla} \times [a(x, y)\hat{\boldsymbol{z}}], \quad (3)$$

where $a\hat{\boldsymbol{z}}$ is the vector potential for the two-dimensional component of turbulence and $a(x, y)$ can be called the potential function. The arbitrary constant in the vector potential is chosen so that $\langle a \rangle = 0$.

For pure two-dimensional turbulence, with no slab component, magnetic field lines can remain trapped near certain (x, y)-coordinates because they always follow contours of constant a. As an example, Figure 1 shows a contour plot of $a(x, y)$ for a specific representation of two-dimensional turbulence that was generated to have desired statistical properties, as will be discussed in detail in § 3. The circles in Figure 1 indicate O-points [local maxima or minima in $a(x, y)$] where the contours remain trapped within "islands" of the two-dimensional turbulence (or filaments in three-dimensional space). We also indicate X-points, i.e., saddle points of $a(x, y)$. This is an example of how turbulence with homogeneous statistical properties can have small-scale topological structure.

The ensemble average statistics of the field line random walk were calculated by Matthaeus et al. (1995). A diffusion coefficient, D, is defined by $\langle \Delta x^2 \rangle = 2D\Delta z$, where Δx is the change in a perpendicular coordinate over a distance Δz along the mean field. Each turbulence component is associated with a value of D; the overall value is $D = D_{\rm slab}/2 + [(D_{\rm slab}/2)^2 + (D_{\rm 2D})^2]^{1/2}$. Under normal solar wind conditions, $D_{\rm slab}$ is very small ($\approx 5 \times 10^{-4}$ AU). The total diffusion coefficient can be estimated from the *IMP-8* and *Ulysses* data sets.

For most solar events shown by McKibben et al. (2001), the 30–70 MeV proton time-intensity profiles at the two spacecraft are very similar, in shape as well as in absolute magnitude, immediately after the peak in particle intensity. Only the event of 2000 November 8 shows a distinctly diffusive rise at *Ulysses* before matching *IMP-8* data in the decay phase. Therefore, we have fitted this most diffusive event, using a Reid profile (Reid 1964) centered at the Archimedean field line of the flare site at the radial distance of *Ulysses* (2.35 AU), to provide a lower bound on the particle diffusion coefficient $\kappa_\perp$. Based on this conservative estimate, the *IMP-8* and *Ulysses* observations

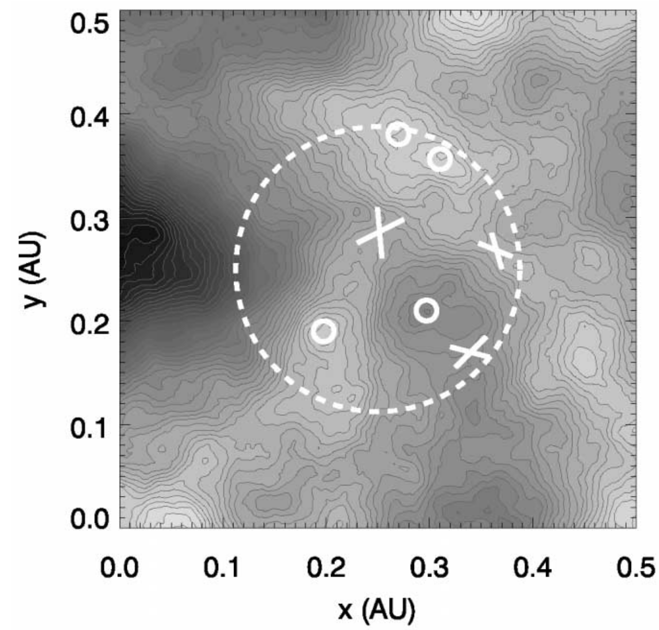

FIG. 1.—Contour plot of the potential function for a representation of the two-dimensional turbulence component. The magnetic field due to this component follows contours of constant potential. Field lines near O-points are trapped within topological "islands," while field lines near X-points or outside islands rapidly travel to other locations. In the solar wind, magnetic field lines undergo an additional random walk as a result of the slab component of turbulence, which allows them to eventually escape from islands surrounding O-points.

require an SEP perpendicular diffusion coefficient of $\kappa_\perp \geq 1.3 \times 10^{21}$ cm^2 s^{-1}, or $\kappa_\perp/\beta \geq 4 \times 10^{21}$ cm^2 s^{-1}, where β is the particle speed divided by the speed of light. This is of the same order of magnitude as previous estimates (e.g., Parker 1963; Palmer 1982). Using the field line random walk concept (Jokipii 1966), which in itself yields an underestimate of D (Matthaeus et al. 2003), one obtains a total field line diffusion coefficient of $D > 0.02$ AU, which is much greater than $D_{\rm slab}$. Conversely, since the model of Giacalone et al. (2000) uses slablike fluctuations (in the sense that fluctuations propagate in z), their model yields insufficient field line diffusion to explain the *IMP-8*/*Ulysses* observations.

Therefore, the two-dimensional component of turbulence dominates the ensemble average field line diffusion, and $\Delta x_{\rm rms} = \langle \Delta x^2 \rangle^{1/2} > 0.2$ AU at Earth orbit. However, such ensemble average statistics cannot apply to observations of dropouts because the dropouts correspond to a filamentation over ~0.03 AU, which would be completely washed out by such rapid diffusion.

3. CONDITIONAL STATISTICS

Instead of ensemble average statistics, let us now consider conditional statistics, depending on the initial location of a magnetic field line. If a field line is near an O-point, within an island of the two-dimensional turbulence (see Fig. 1), the two-dimensional contribution to the random walk is suppressed. The field line is temporarily trapped, with diffusion at the much slower rate characteristic of slab turbulence. On the other hand, magnetic field lines that start outside islands are rapidly carried far away by the two-dimensional turbulence.

In particular, suppose that particles are injected in a spatially

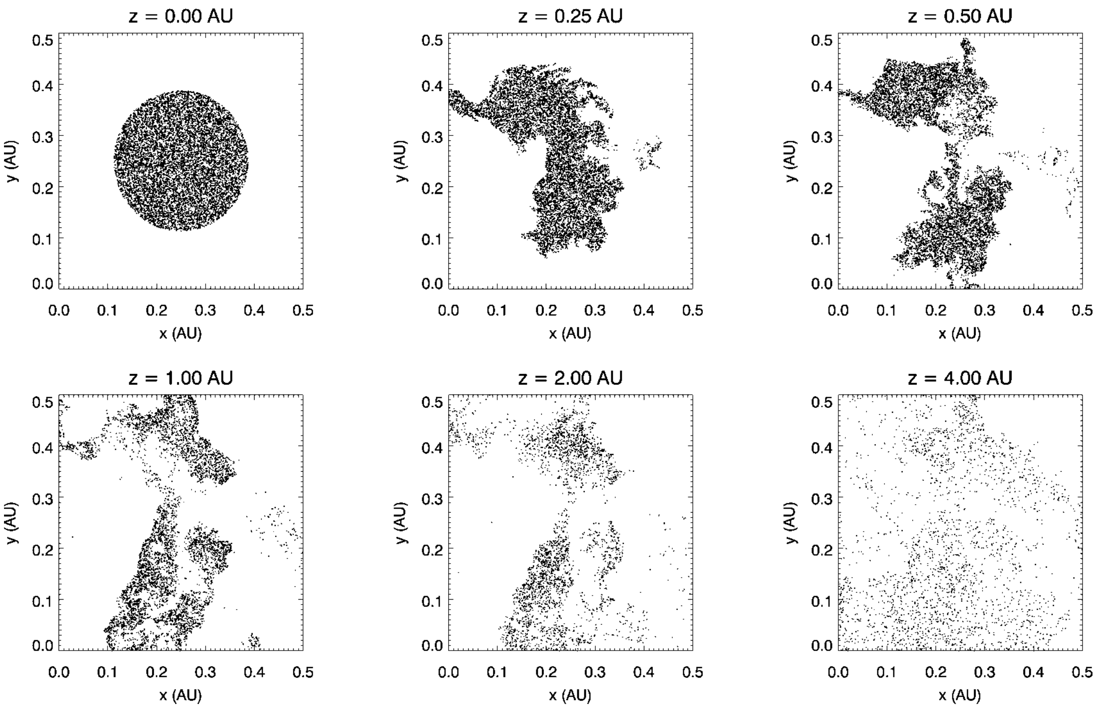

FIG. 2.—Scatter plot of the locations of magnetic field lines that are initially (at $z = 0$) located within a circle (simulating the region where particles are injected as a result of an impulsive solar flare). At intermediate-z values, field lines within islands of the two-dimensional turbulence (around the O-points shown in Fig. 1) remain trapped, while field lines in other regions spread rapidly. This explains the filamentary distribution of particles as indicated by dropout features. At large z values, all field lines diffuse rapidly, which explains the *Ulysses* and *IMP-8* observations of SEP diffusion throughout the inner solar system.

localized region, say, a circle of radius ρ. Then $z_1 = \rho^2/(4D)$ is a characteristic distance over which field lines outside islands diffuse out of the circle. If an island has diameter d, then $z_2 = d^2/(16D_{\rm slab})$ is the typical distance along the mean field over which field lines escape from the island, given diffusion due to the slab component. If slab diffusion is weak, we can have $z_1 < z_{\rm obs} < z_2$, where $z_{\rm obs}$ is the distance of the observer. We suggest that dropouts are observed under these conditions. Magnetic field lines (and the low-energy particles orbiting them) that start deep within islands mostly remain trapped, while those outside the islands rapidly escape from the injection region, leaving gaps with a low density of particles. On the other hand, after a long distance ($z_{\rm obs} > z_2$), essentially all field lines have escaped their temporary topological traps, corresponding to a rapid lateral diffusion of field lines (with the ensemble average diffusion coefficient D) and of particles.

This idea is confirmed by computer simulations that trace field line trajectories in representations of two-dimensional+slab turbulence for typical solar wind values, using Cartesian geometry for simplicity. The field line random walk is then a surrogate for particle gyrocenter motion. The simulations involve two steps:

1. The first step involves generating representations of slab and two-dimensional turbulence with desired statistical properties, such as the observed Kolmogorov power-law spectrum over the inertial wavenumber range (Jokipii & Coleman 1968), using the power spectrum of Ruffolo & Matthaeus (2003) and random phases, and then using inverse fast Fourier transforms to obtain $\boldsymbol{b}_{\rm slab}(z)$ and $\boldsymbol{b}_{\rm 2D}(x, y)$. The transform in z used 2^{22} ($\approx 4.2 \times 10^6$) points, representing a length of 25 AU. The transform in x and y used 2048 points in each dimension, corresponding to a length of 2.5 AU. The simulations in the present work were for parameters believed to correspond to typical solar wind conditions: $b/B_0 = 0.5$, a fraction of slab turbulent energy $f_s = 0.2$ (following Bieber et al. 1994), a slab correlation length $\ell_c = 0.02$ AU, and an ultrascale of two-dimensional turbulence $\tilde{\lambda} \equiv (\langle a^2\rangle/\langle b^2\rangle_{\rm 2D})^{1/2} = 0.06$ AU. The ultrascale is believed to roughly correspond to the size of the largest islands, and this value corresponds to $D = 0.02$ AU, as a conservative lower limit. This lower limit is consistent with a previous estimation of $\tilde{\lambda}$ (Matthaeus, Smith, & Bieber 1999).

2. The second step involves tracing magnetic field lines, i.e., solving the coupled ordinary differential equations

$$\frac{dx}{dz} = \frac{b_x(x, y, z)}{B_0}, \quad \frac{dy}{dz} = \frac{b_y(x, y, z)}{B_0}. \tag{4}$$

We use a fourth-order Runge-Kutta method with adaptive time stepping regulated by a fifth-order error estimate step (Press et al. 1992).

Our computer simulations for several representations (several sets of random phases), different values of $\tilde{\lambda}$, and different spectral forms at low wavenumber yielded qualitatively similar results, including structures corresponding to dropouts of ~0.03 AU as in the *ACE* observations.

The results shown in Figure 2 demonstrate the behavior described above. The upper left-hand panel shows random initial locations within a circle, corresponding to the injection region where field lines are populated with SEPs. Field lines are then traced from those initial locations as a function of z. The subsequent panels, cross sections at longer distances along the mean field, show filamentary structures in the distribution of SEPs. A spacecraft near Earth ($z \approx 1$ AU) samples a transept through this highly inhomogeneous distribution. The simulation results are consistent with observed dropouts of ∼0.03 AU. At longer distances, essentially all field lines (and particles) have diffused away, leading to the rapid propagation of particles throughout the inner heliosphere at later times.

4. DISCUSSION AND CONCLUSIONS

Note that we identify dropouts with topological structures that develop in solar wind turbulence, not with initial motions at the solar surface. We see the effects of islands of various sizes d due to the self-similar nature of turbulence, including islands within islands, but those much wider than ρ do not confine particles near the injection region.

For a wide injection region, $z_1 > z_{\rm obs}$, and dropouts should not be seen. Indeed, another class of solar events, gradual flare/coronal mass ejection events, inject particles over a much wider region (Reames 1990) and do not exhibit dropouts (see the similar argument of Mazur et al. 2000 and Giacalone et al. 2000). We confirm that our model explains the lack of dropouts for gradual events by a "control run" in which field lines are randomly distributed throughout the simulation region. The distribution indeed remains uniformly random at all distances (which in the context of our model is required by Liouville's theorem).

This new view of the perpendicular transport of energetic particles in space plasmas can also reconcile another pair of apparently conflicting observations. Impulsive solar events selected for a strong SEP electron increase were shown to have a narrow distribution in solar longitude (Reames et al. 1990). This indicates only limited lateral spreading for the bulk of SEPs, which we attribute to trapping within small-scale topological islands, representing a "core" region of high particle density (Fig. 3). On the other hand, recent spacecraft observations of type III radio bursts and associated SEPs indicate that SEP electrons and ions can undergo broad lateral motion (up to ∼90° in solar longitude) during their transport from the Sun to Earth orbit (Cane & Erickson 2003). In our view, this laterally extended but less intense "halo" of SEPs corresponds to particles on field lines initially located outside local islands of two-dimensional turbulence. (Note that particles observed by *Ulysses* do not necessarily correspond to this halo since they may have undergone lateral diffusion beyond 1 AU.) Indeed, the absence of these halo SEPs from the core region is manifest as dropouts.

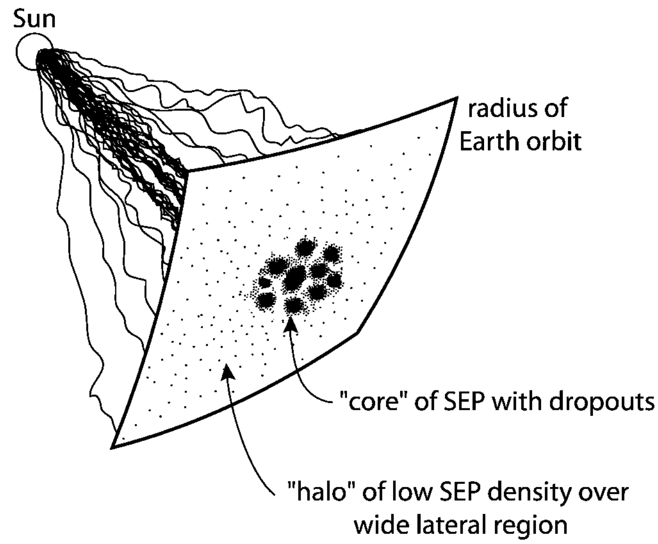

FIG. 3.—Illustration of interplanetary magnetic field lines populated with SEPs from a localized source region near the Sun, as expected for an impulsive solar flare. In the two-dimensional+slab model of solar wind turbulence, some field lines are trapped in filaments corresponding to the small-scale topology, i.e., islands of the two-dimensional turbulence, out to Earth orbit, while interstitial field lines spread laterally to large angular distances. This leads to the observed "core" region of SEPs with dropouts and an extended "halo" region.

Finally, we note that the problem considered here is directly analogous to the Hamiltonian flow of a dynamical system (in two-dimensional phase space) with time-dependent, random forcing (on the substitutions $a \rightarrow H$ and $z \rightarrow t$). Therefore, our qualitative conclusions apply to such systems in general.

The authors acknowledge useful discussions with Hilary Cane, Joe Dwyer, and John Bieber. This research was partially supported by a Basic Research Grant and a Royal Golden Jubilee Fellowship from the Thailand Research Fund, the Rachadapisek Sompoj Fund of Chulalongkorn University, and the NASA Sun-Earth Connections Theory Program (grant NAG5-8134).

REFERENCES

Bieber, J. W., Matthaeus, W. H., Smith, C. W., Wanner, W., Kallenrode, M.-B., & Wibberenz, G. 1994, ApJ, 420, 294
Bieber, J. W., Wanner, W., & Matthaeus, W. H. 1996, J. Geophys. Res., 101, 2511
Cane, H. V., & Erickson, W. C. 2003, J. Geophys. Res., 108(A5), SSH 8-1
Dröge, W. 2000, Space Sci. Rev., 93, 121
Dröge, W., Wibberenz, G., & Klecker, B. 1990, Proc. 21st Int. Cosmic Ray Conf. (Adelaide), 5, 187
Evenson, P., Meyer, P., & Yanagita, S. 1982, J. Geophys. Res., 87, 625
Giacalone, J., Jokipii, J. R., & Mazur, J. E. 2000, ApJ, 532, L75
Jokipii, J. R. 1966, ApJ, 146, 480
Jokipii, J. R., & Coleman, P. J. 1968, J. Geophys. Res., 73, 5495
Matthaeus, W. H., Goldstein, M. L., & Roberts, D. A. 1990, J. Geophys. Res., 95, 20,673
Matthaeus, W. H., Gray, P. C., Pontius, D. H., Jr., & Bieber, J. W. 1995, Phys. Rev. Lett., 75, 2136
Matthaeus, W. H., Qin, G., Bieber, J. W., & Zank, G. P. 2003, ApJ, 590, L53
Matthaeus, W. H., Smith, C. W., & Bieber, J. W. 1999, in AIP Conf. Proc. 471, Solar Wind Nine, ed. S. Habbal, R. Esser, J. V. Hollweg, & P. A. Isenberg (Woodbury: AIP), 511
Mazur, J. E., Mason, G. M., Dwyer, J. R., Giacalone, J., Jokipii, J. R., & Stone, E. C. 2000, ApJ, 532, L79
McKibben, R. B. 1972, J. Geophys. Res., 77, 3959
McKibben, R. B., Lopate, C., & Zhang, M. 2001, Space Sci. Rev., 97, 257
Palmer, I. D. 1982, Rev. Geophys. Space Phys., 20, 335
Parker, E. N. 1958, ApJ, 128, 664
———. 1963, Interplanetary Dynamical Processes (New York: Wiley-Interscience)
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in FORTRAN: The Art of Scientific Computing (Cambridge: Cambridge Univ. Press)
Reames, D. V. 1990, ApJ, 358, L63
Reames, D. V., Cane, H. V., & von Rosenvinge, T. T. 1990, ApJ, 357, 259
Reid, G. C. 1964, J. Geophys. Res., 69, 2659
Ruffolo, D., & Matthaeus, W. H. 2003, ApJ, submitted
Sanderson, T. R., Erdös, G., Balogh, A., Forsyth, R. J., Marsden, R. G., Gosling, J. T., Phillips, J. L., & Tranquille, C. 2000, J. Geophys. Res., 105, 18,275
Scholer, M., & Morfill, G. 1975, Sol. Phys., 45, 227